

## Recent Advances in Accuracy Enhancement of MEMS-based IMUs for Navigation Applications

**Aboelmagd Noureldin, Mark Eberts, Craig Johnston**  
NavINST – Navigation & Instrumentation Research Laboratory  
Department of Electrical and Computer Engineering  
Royal Military College of Canada  
Kingston, Ontario, Canada

**Naser El-Sheimy**  
MMS – Mobile Multi-Sensor Research Group  
Department of Geomatics Engineering  
The University of Calgary  
Calgary, Alberta, Canada

**Jeff Bird**  
DRDC – Defence Research and Development Canada, Ottawa  
Department of National Defence  
Ottawa, Ontario, Canada

### **ABSTRACT**

*Many land vehicles are presently equipped with on-board navigation systems that currently rely entirely on the availability and accuracy of GPS. The accessibility of low cost MEMS-based inertial measurement units (IMU) can be used to provide precise position and velocity data for GPS signal acquisition and reacquisition after outages. This reduces the time and the search domain required for detecting and correcting cycle slips.*

*Providing reliable and accurate navigation solution during GPS outages at low cost necessitates enhancing the performance of MEMS-based inertial sensors with robust de-noising technique that extracts the motion dynamics from the sensor errors prior to INS mechanization and INS/GPS integration. In addition, robust MEMS-based IMU/GPS data fusion requires modifying the conventional integration techniques, predominantly based on Kalman filtering (KF), so that they account for the shortcomings of low cost MEMS-based inertial sensors.*

*This paper summarizes our recent results and analysis on: (1) the impact of de-noising MEMS-based inertial sensors on the overall performance; (2) the effect of using a segmented KF module to fuse MEMS-based INS with GPS; (3) examining the benefits of closed loop IMU/GPS integration architecture; (4) the value of stochastic modeling of MEMS-based inertial sensor errors.*

### **1.0 INTRODUCTION**

The last two decades have shown an increasing trend in the use of positioning and navigation technologies in land vehicle applications. Most of the current vehicular navigation systems rely on the global positioning

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system (GPS) that is capable of providing accurate position and velocity information. To be able to provide such accurate measurements, GPS needs at least four satellites with good geometry. In addition, there must be a direct line-of-sight between the GPS antenna and those satellites. Unfortunately this is not always possible since a GPS signal may be lost when driving around obstacles (through urban areas, overpasses or tunnels on highways, tree-lined streets, etc.), or when operating in poor weather conditions. The satellite signal blockage results in deterioration of the overall position accuracy. On the other hand, the inertial navigation system (INS) is a self-contained system. It incorporates three orthogonal accelerometers and three orthogonal gyroscopes, which measure three linear accelerations and three angular rates respectively. A set of mathematical transformations and integrations with respect to time, known as the mechanization equation, are applied to the raw measurements from the INS sensors to determine position, velocity and attitude information. Unfortunately, INS cannot replace the GPS or operate on a stand-alone basis. During the mechanization procedure, the accuracy of INS position components deteriorates with time due to the inherent sensor errors that exhibit considerable long-term growth. These errors include white noise, correlated random noise, bias instability, and angle random walk (IEEE Std. #647, 1995). The errors are stochastic in nature and can cause a significant degradation in the INS performance over a long period of operation.

In order to overcome the problems associated with the operation of GPS and INS on their own, the two systems are often paired together so that the drawbacks associated with each system are eliminated. The INS/GPS data fusion is commonly performed using Kalman Filter (KF) [1]. This method requires a dynamic model of both INS and GPS errors, a stochastic model of the inertial sensor errors, and *a priori* information about the covariance of the data provided by both systems [2]. Since GPS has a consistent, long-term accuracy, it is used to update both INS position and velocity components and thus to prevent the long-term growth of their errors. On the other hand, the accurate short-term information provided by the INS is also used to solve problems related to GPS like cycle slips and clock biases. Should a GPS outage occur, KF operates in prediction mode, correcting the INS information based on the system error model.

The objective of this research is to provide a reliable MEMS-based INS/GPS positioning module that enhances the overall system accuracy and enables robust and accurate positioning information during GPS outages. Motivated by this objective, this research focuses on new techniques for de-noising MEMS-based inertial sensors and implementing a closed loop KF utilizing stochastic modeling of MEMS-based inertial sensor errors and segmentation to improve positional information during GPS outages.

## 2.0 METHODOLOGY

### 2.1 De-Noising MEMS-Based Inertial Sensors

In order to achieve the most accurate INS solution it is widely accepted that pre-filtering or de-noising, each of the inertial sensor signal be carried out prior to integration with the GPS data. For MEMS-based inertial sensors, efficient de-noising prior to mechanization and INS/GPS integration enhances the overall performance significantly. Optimal LPF and wavelet de-noising (WDN) techniques are the state of the art methods used to eliminate or minimize short-term errors from inertial sensor signals. However, all of these techniques have had limited success at removing the long-term errors that are combined with the true motion vehicle dynamics that appear at the low end of the frequency spectrum. In addition, they have not showed an increase in performance when applied to low cost MEMS-based IMUs.

## 2.2 Spectral De-Noising

In order to remove the correlated inertial sensor noise existing at the low frequency band, it is required to have a technique that can remove the long-term errors located at the low frequency band. In order to achieve this it would be beneficial to obtain a frequency resolution that is greater than that of the FFT. This would allow for a very detailed analysis of the inertial sensor data at the low end of the frequency spectrum. Traditional methods, such as the FFT, WMRA and WPT are not able to provide high-resolution spectral analysis.

Korenberg [3], developed a technique known as the Fast Orthogonal Search (FOS) that has proven capable of conducting high-resolution spectral analysis [4], and is also able to detect frequencies of a time-series buried in noise [1]. FOS has been applied to inertial sensor pre-filtering with success [5].

FOS is a general-purpose non-linear modeling technique that can be applied to spectral estimation and time-frequency estimation. Derivations of the FOS algorithm and an in-depth explanation of FOS may be found in [3].

## 2.3 Application of FOS to Inertial Sensor Pre-Processing

During this research, FOS was used to improve the accuracy of the inertial sensors contained within the INS. FOS is capable of detecting frequencies that are buried in coloured or white noise, detecting sub-harmonic frequencies and at performing high-resolution spectral analysis. This makes it an ideal algorithm for application to MEMS-based inertial sensors. The algorithm chosen to model and synthesize a noise free estimate of vehicle motion dynamics is known as Short-Time FOS (ST-FOS) and is represented in Figure 1.

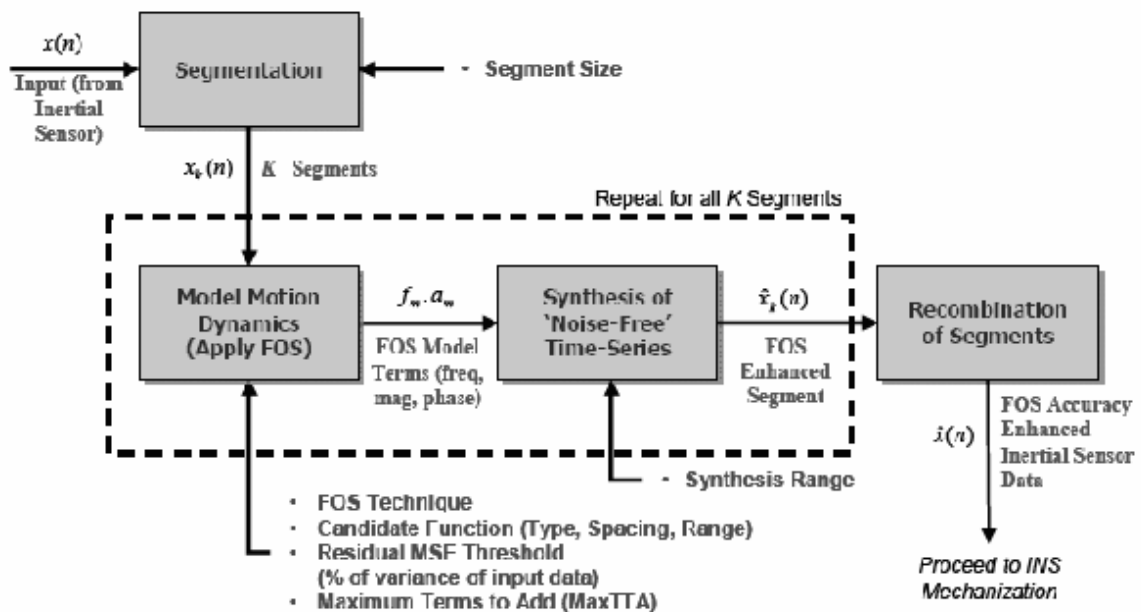


Figure 1. Block Diagram of FOS Spectral De-noising [5].

The flow of FOS de-noising is such that a noisy segment of data,  $x(n)$ , representing one of the six MEMS inertial sensor outputs is segmented into non-overlapping segments. Typically, each segment represents 2 to 5 seconds worth of information and can therefore be treated as stationary data, from statistical point of view.

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FOS is then performed on each segment to reduce the noise and detect the vehicle dynamics. The stopping criteria for FOS are critical components of the algorithm as without them, FOS may include unwanted noise terms as part of the final model. The outputs of the FOS algorithm provide information on the frequency, magnitude and phase content of each segment. Once FOS has completed its analysis of each segment, they are recombined to provide the overall noise-free time series of the input signal. Finally, the FOS de-noised inertial sensor data goes through the INS mechanization process, and then through the Kalman filter that provides the final augmented navigation solution.

### 2.4 Kalman Filtering Integration

KFs are routinely used in applications where it is needed to combine noisy sensor outputs to estimate the state of a system with uncertain dynamics. With its long history, the KF is by far the most widely used technique for integrating GPS and INS. It is considered to be the benchmark for all new techniques to reach and exceed.

When INS is combined with GPS in a KF the system an increase in performance that is greater than that of the individual systems. The KF enables the INS to be updated using GPS information and the INS can provide a navigation solution when there is a GPS outage.

#### 2.4.1 Open Loop and Closed Loop Mechanization

The mechanization process can be carried out in either an open loop or a closed loop. In an Open Loop configuration the process is computed without correcting any of the navigation parameters (position, velocity and attitude) before being fed into the loop (Figure 2). In other words, open loop approach leaves the position, velocity and attitude errors to be accumulated. This approach assumes that the dynamic model of INS errors can provide reliable estimates of the accumulated errors of the position, velocity, and attitude when operated by the KF. In a Closed Loop configuration the navigation parameters are corrected for the corresponding errors before being computed and fed into the next epoch (Figure 3). \*(k-1 denotes previous value)

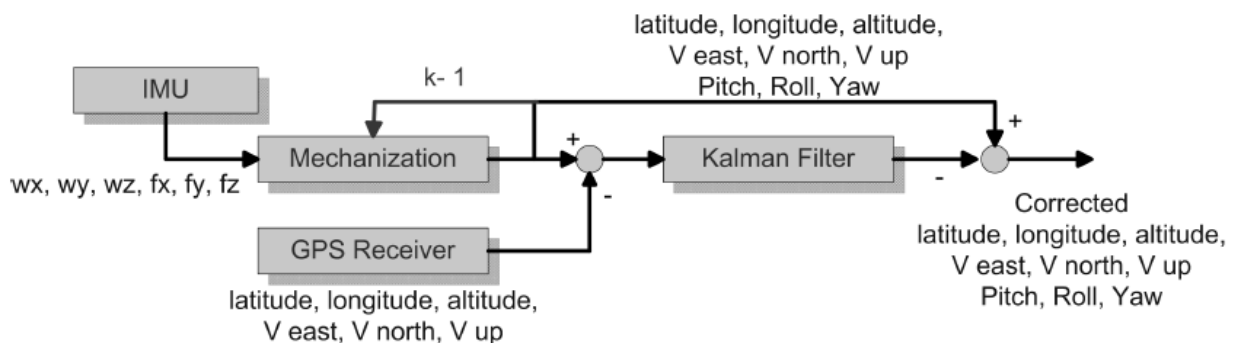


Figure 2. Block Diagram of Open Loop IMU Mechanization.

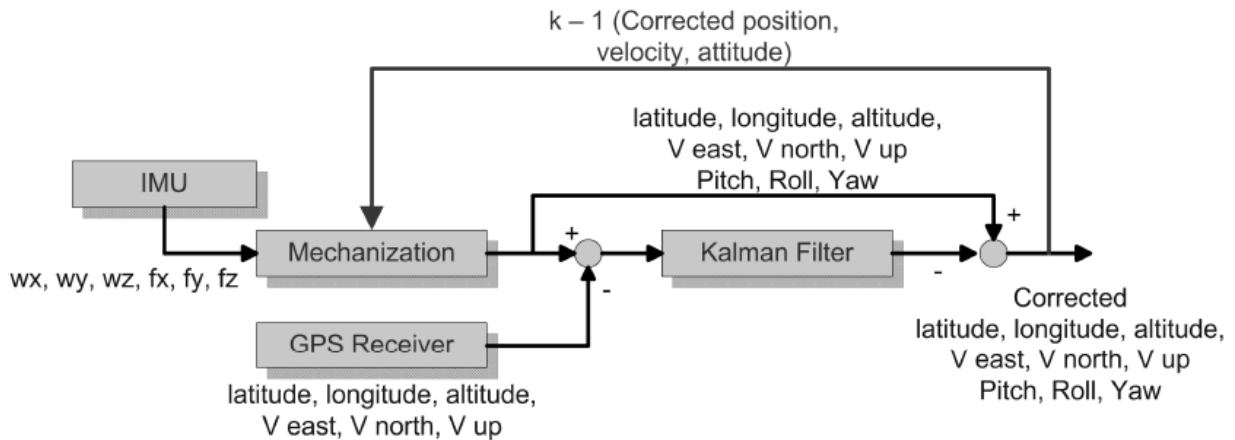


Figure 3. Block Diagram of Closed Loop IMU Mechanization.

### 2.4.2 Segmenting the Kalman Filter

The KF is a recursive algorithm, meaning the Prediction and update stages can continue as long as new data is available. However, practical limits under certain conditions can cause divergence in the filter [6]. Segmentation of the KF can help to re-initialize some or all of the filter parameters. These divergencies are more commonly caused by roundoff errors, modeling errors or observability problems [6].

When using lower cost MEMS sensors for inertial navigation the stand alone solution may lead to fast growth of position, velocity and attitude errors if left uncorrected. When it is integrated with GPS, among other benefits, the output can now be corrected for position and velocity. For pitch, roll and azimuth, the KF can only correct for the partially observable errors. There is no direct correction of the INS attitude components from the GPS updates and this can lead to error growth.

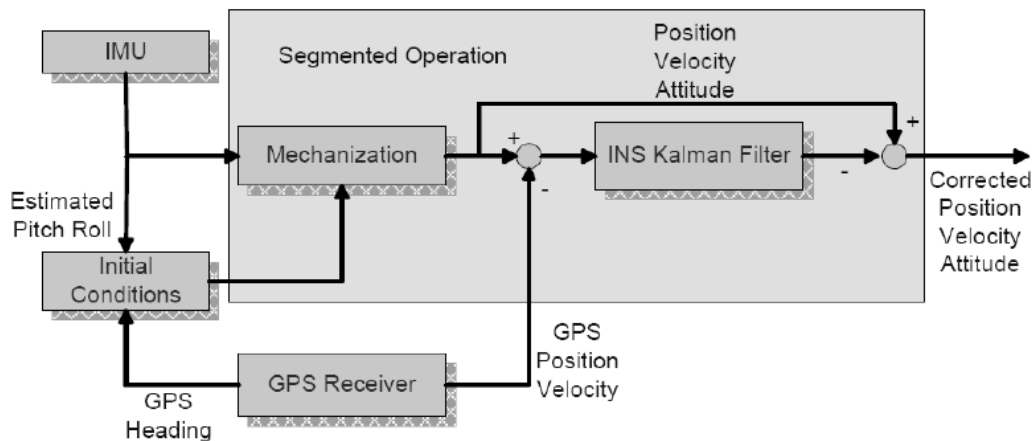


Figure 4. Block diagram of segmented KF.

To help the MEMS IMU correct the attitude, the INS Mechanization and KF operate in segments (Figure 4). The advantage of segmentation is the ability to reset the attitude components and feed these values to the INS

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mechanization process as new initial values. This helps to bound the error growth but it also can be a poor estimation of the true attitude. For proper estimation of the pitch and roll the assumption has to be made that the vehicle is at constant velocity and there is no acceleration. It can now be assumed that any acceleration would be due to gravity and the pitch and roll can be determined by their measurement components of the gravity vector in the x and y accelerometers. The azimuth can be estimated from GPS heading output. The equations for the pitch and roll update are:

$$Pitch_{update} = \arcsin \frac{f_y}{g} \quad (1)$$

$$Roll_{update} = \arcsin \frac{f_x}{g} \quad (2)$$

Where  $f_x$  and  $f_y$  are the accelerometer measurements ( $m/sec^2$ ) in the x and y directions and g the acceleration due to gravity ( $m/sec^2$ ). The azimuth is updated only when the GPS heading output is available.

## 2.5 Stochastic Modeling of Inertial Sensor Errors

### 2.5.1 The Gauss-Markov Process

In most KF implementations the sensor noise portion of the State Transition Matrix  $F$  is based on a 1<sup>st</sup> order Gauss Markov model. The GM process is frequently used in engineering applications because of its simplicity and partly because the designer often lacks the detailed knowledge of the process under consideration that is needed to devise a more complicated model [6].

### 2.5.2 The Autoregressive Model

The assumption of sensor random errors following the stochastic nature of a first order GM model is not always valid, especially for MEMS inertial sensors. When we carefully examined the autocorrelation sequences of several MEMS-based inertial sensors, we were able to determine that the random errors associated with these sensors are different from that of a 1<sup>st</sup> order GM process and it would be more appropriate to be described by high order GM processes.

To avoid the problem of inaccurate error models another method for modeling inertial sensor errors can be used. Stochastic modeling of inertial sensor errors can be achieved by modeling these errors as higher order autoregressive (AR) processes, and obtaining the AR model parameters adaptively. Stationary sensor data can be used while computing the coefficients of a higher order AR model such as the Burg method [7]. Fitting stationary experimental data to an AR model can result in a better stochastic model than assuming the sensor conforms to a 1<sup>st</sup> order GM model. The  $p^{th}$  order autoregressive model can be described as:

$$y(k) = -\alpha_1 y(k-1) - \alpha_2 y(k-2) - \dots - \alpha_p y(k-p) + \beta_0 w(k) \quad (3)$$

where  $\alpha_1$ ,  $\alpha_2$ ,  $\dots$ ,  $\alpha_p$  and  $\beta_0$  are the model coefficient parameters.

Since the AR model is going to be applied to all six inertial sensors, each increase in the model order will lead to six more states added to the error state vector. For this reason the lowest possible order must be chosen so that the root of the mean square error (RMSE) of the model converges and a lower order model does not

complicate the KF procedure. Equation (3) shows the  $p^{th}$  order autoregressive equation. These values are higher order and must be reduced to a set of first order equations before they can be implemented into the KF. Figure 5 shows the RMSE versus increasing AR model orders when Burg methods were used to obtain the AR model parameters for the Crossbow MEMS based accelerometers.

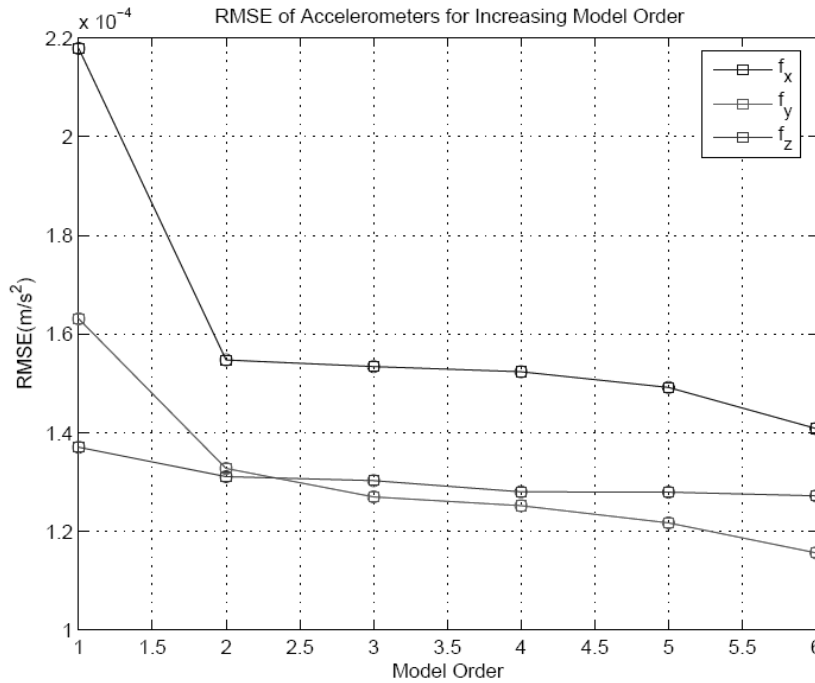


Figure 5. Crossbow MEMS IMU Accelerometer Prediction RMSE.

### 2.5.3 AR Model Parameters

In order to keep the INS error model simple and to facilitate the KF procedure, a 2<sup>nd</sup> order AR model was used for each inertial sensor. A second order model will have two coefficients to substitute in for  $\alpha_1$  and  $\alpha_2$  of Equation (3). Resulting in:

$$y(k) = -\alpha_1 y(k-1) - \alpha_2 y(k-2) + \beta_0 w(k) \tag{4}$$

To reduce Equation (4) to a set of first order equations it must be rearranged to the form:

$$\begin{bmatrix} Y_{k-1} \\ Y_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha_2 & -\alpha_1 \end{bmatrix} \begin{bmatrix} Y_{k-2} \\ Y_{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_0 \end{bmatrix} w_k \tag{5}$$

Apparently, each inertial sensor will be modeled stochastically using two state variables and when augmented to the INS error model, it will take the form:



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$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}_k = \begin{bmatrix} 0 & 1 \\ -\alpha_2 & -\alpha_1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}_{k-1} + \begin{bmatrix} 0 \\ \beta_0 \end{bmatrix} w_k \quad (6)$$

In Equation (6) for one sensor, a second order model produces two state variables  $Y_1$  and  $Y_2$  and two coefficients that describe the model  $\alpha_1$  and  $\alpha_2$ . Using this AR 2<sup>nd</sup> order error model, the KF state space would increase to 21 variables from 15 and the Transition Matrix  $F$  would include the 12 new coefficients and the  $\beta^d$  and  $\beta^b$  values would be removed since the first order GM process is no longer needed.

### 3.0 RESULTS

#### 3.1 Impact of Stochastic Modeling

The impact of the 2<sup>nd</sup> order AR model on the overall positioning accuracy will be examined and compared to the conventional method using a 1<sup>st</sup> order GM model during nine GPS outages intentionally introduced into the trajectory. Figure 6 shows the maximum position error during the nine GPS outages for the two cases of using the 2<sup>nd</sup> order AR model and the conventional 1<sup>st</sup> order GM error model. It can be determined from this figure that higher positioning accuracy can be achieved during 30-second GPS outages when AR stochastic models are adopted. Except for the two cases (GPS outages 2 and 7) the position errors can be reduced by utilizing AR models. Apparently, the 2<sup>nd</sup> order stochastic error models of MEMS-based inertial sensors benefit the positioning accuracy in cases where the overall system tends to provide large position errors during GPS outages. The high errors of GPS outages 6 and 8 were shown to occur when a GPS outage began immediately prior to a change in vehicle dynamics such as a sharp turn or stop and go traffic. These high errors are likely caused by the non-linearity of the vehicle dynamics. It is worthy mentioning that the results shown in Figure 5 was obtained without performing any inertial sensor de-noising. Further improvements to the accuracies shown on Figure 5 can be achieved by augmenting the other methods proposed in this research. The impact of these methods when augmented with the AR models will be shown in the following sections.



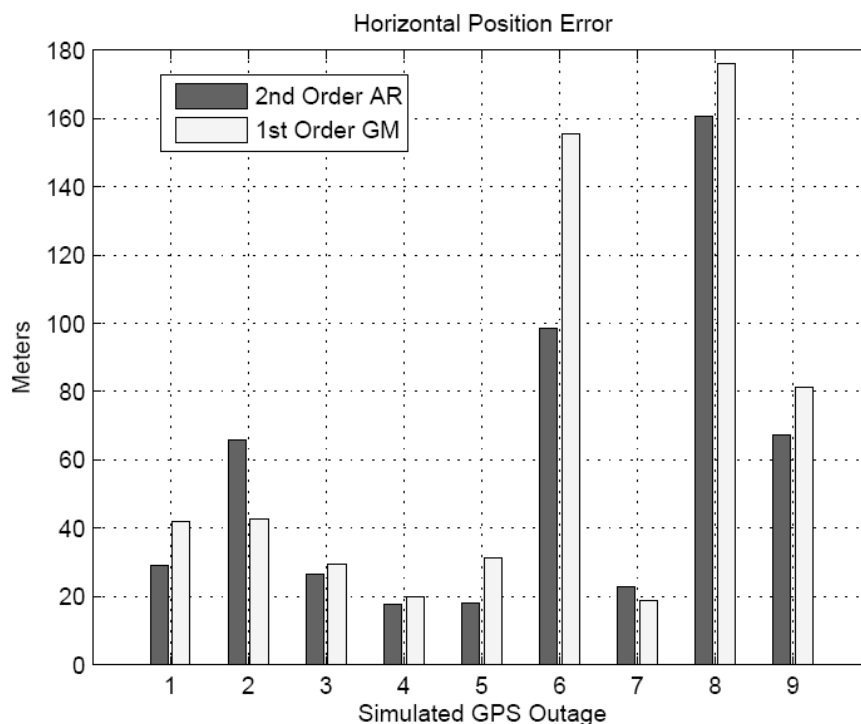


Figure 6. Total position error during 30 second GPS outages while using 1<sup>st</sup> order GM and 2<sup>nd</sup> order AR error models.

### 3.2 Impact of Kalman Filter Segmentation / In-Motion Update

MEMS grade IMUs are notorious for poor attitude performance when left unaided. To improve the performance the KF was segmented so that pitch, roll and azimuth could be updated while in-motion and the error growth can be minimized. KF segmentation was used in all of the cases above since its impact is so pronounced.

### 3.3 Impact of Inertial Sensor De-noising

Inertial sensor de-noising in this study was performed using wavelet de-noising (WDN) and FOS. Comparison between both techniques is shown below. The de-noising parameters for the WDN and FOS methods are provided in Tables 1 and 2, respectively.

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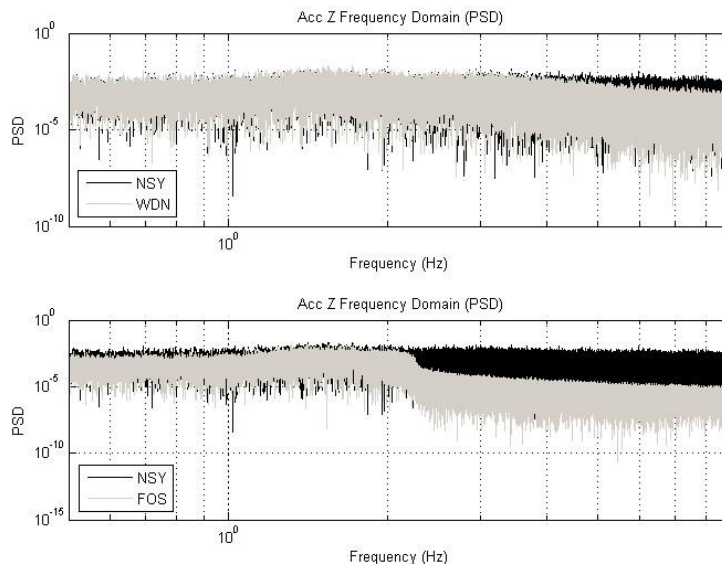
**Table 1. Wavelet De-noising Parameters.**

Signal	Wavelet	LOD	Threshold Parameters
x-gyro, y-gyro	db5	4	Soft, SURE, MLN rescaling
z-gyro	db5	5	Soft, SURE, MLN rescaling
x-acc, y-acc	db5	5	Soft, SURE, MLN rescaling
z-acc	db5	4	Soft, SURE, MLN rescaling

**Table 2. FOS De-Noising Parameters.**

Signal	Technique	Candidate Frequencies (Hz)	Synthesis Range	Window Size (n)	maxTTA	Threshold
x-gyro, y-gyro	FOS	0 - 4 Hz @ 1/10 FFT Res, 4 - fs/2 @ FFT Res	0 - 4 Hz	~400	23	4% var (NSY)
z-gyro	FOS	0 - 2 Hz @ 1/10 FFT Res, 2 - fs/2 @ FFT Res	0 - 2 Hz	~400	23	4% var (NSY)
x-acc, y-acc	FOS	0 - 2 Hz @ 1/10 FFT Res, 2 - fs/2 @ FFT Res	0 - 2 Hz	~400	23	4% var (NSY)
z-acc	FOS	0 - 4 Hz @ 1/10 FFT Res, 4 - fs/2 @ FFT Res	0 - 4 Hz	~400	23	4% var (NSY)

Once the experimental data was collected, it was processed using the parameters in the Tables above. Spectral analysis was completed on the NSY, WDN and FOS data. The results presented in Figure 7 indicate that the WDN de-noising was only successful at removing short-term errors above 5Hz, while FOS removed more sensor errors at the lower frequency ranges (approx above 2.5Hz).



**Figure 7. Spectral Analysis of the NSY, WMRA and FOS data.**

The results of the signal de-noising comparing the NSY data to the WDN and FOS data translated to the position domain are presented in Figure 8. Upon inspection of Figure 8, the difference between the horizontal position error between the NSY and both de-noised signals is significant, however, the FOS signal offers, on average, an even further decrease in the horizontal position error than WDN.

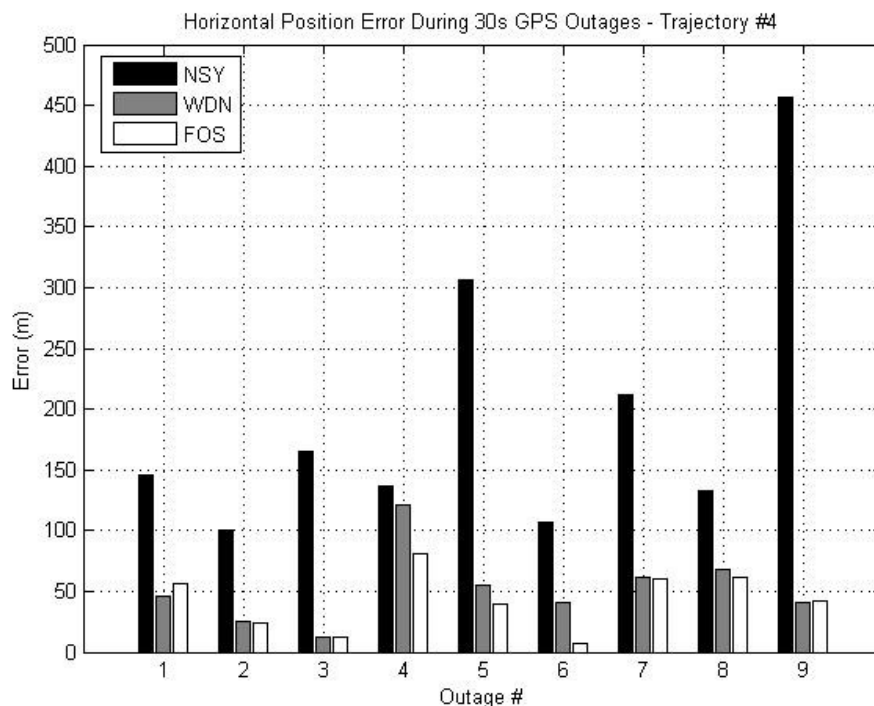


Figure 8. Horizontal position error data for nine 30-second GPS outages.

## CONCLUSION

This paper suggested new techniques for improving an integrated MEMS-based INS/GPS navigation system that can be utilized in numerous applications. We have shown that the utilization of closed loop INS/GPS integration schemes is essential in providing reliable navigation solution when MEMS-based inertial sensors are utilized. The segmentation and the initialization of the attitude angles at the beginning of each data segment improves the overall performances and avoided the growth of both attitude and position errors. We have also demonstrated that the spectral de-noising of inertial sensors using FOS provides superior performance if compared to widely used wavelet de-noising.

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